Lecture 5 Summary

Tues 05/08/14

Vocabulary

* topological space; topology
* open set
* closed set
* open ball
* neighbourhood
* neighbourhood filter
* interior
* closure
* interior point
* close point / adherent point

Examples

TOPOLOGIES

(1) Let X be a set.

The discrete topology on X is:

$$\tau = \{\text{all subsets of } X\}$$

(2) Let (X, d) be a metric space.

The metric space topology on X is:

$$\tau = \{U \subseteq X \mid U \text{ is a union of open balls}\}$$

(3) Let (X, τ) be a topological space. Let $Y \subseteq X$.

The subspace topology on Y is:

$$\tau_Y = \{U \cap Y \mid U \in \tau\}$$

(4) Let (X, τ_X), (Y, τ_Y) be topological spaces.

The product topology on $X \times Y$ is:

$$\tau_{X \times Y} = \{U \subseteq X \times Y \mid U \text{ is the union of } A \times B, \text{ with } A \in \tau_X, B \in \tau_Y\}$$

OPEN & CLOSED SETS

Let $X = \mathbb{R}$ with the metric given by:

$$d(x, y) = |x - y|$$

and the metric space topology.

(1) $(a, b) = \{x \in \mathbb{R} \mid a < x < b\} = B_{\frac{b-a}{2}}\left(\frac{a+b}{2}\right)$ is open.

(2) $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$ is closed.

(3) $[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$ is not open, not closed.

(4) $\emptyset$ and $\mathbb{R}$ are both open and closed.

Homework

- Show that examples (1) – (4) really define topologies

7mm A4